Commercial Tools

Camtasia Studio
http://www.techsmith.com/camtasia.asp

ScreenCam
http://www.smartguyz.com

HyperCam
http://www.hyperionics.com/hc

My Screen Recorder Pro
http://www.deskshare.com/msrpro.aspx

WebEQ
http://www.dessci.com/en/products/webeq

MapleNet
http://www.maplesoft.com/products/maplenet

TurnItIn
http://www.turnitin.com

Journals

Online Journal of Distance Learning Administration
http://www.westga.edu/~distance/jmain11.html

The American Journal of Distance Education
http://www.ajde.com/

Journal of Distance Education
http://www.lib.unb.ca/Texts/JDE/

The Turkish Online Journal of Distance Education
http://tojde.anadolu.edu.tr/

Journal of Library Services for Distance Education
http://www.westga.edu/~library/jlsde/

A Football Play-Calling Experiment to Illustrate the Mixed Strategy Nash Equilibrium

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The determination of the mixed strategy Nash equilibrium is often difficult for students. Further, the understanding of when a player should mix strategies relative to when more simple choices could be made often confuses students. In this paper, I discuss a classroom game that focuses on mixed strategies. Playing the role of a football coach, students choose either to run or pass (if they are an offensive football coach) or to defend the run or defend the pass (if they are a defensive football coach). The game helps provide students an intuitive understanding of the mixed strategy Nash equilibrium, including why and when mixed strategies should be employed.

Key Words: Mixed Strategy Nash Equilibrium, Classroom Games
Disciplines of Interest: Economics

INTRODUCTION

Mixed strategies are covered in almost all game theory courses, in some other (micro) economics courses, and in some business strategy courses. Many games, including commonly discussed games like the confrontations between pitchers and hitters or the battle of the sexes (McCain), require students to determine the optimal play using mixed strategies. One problem with this concept is that students often find determining the mixed strategy Nash equilibrium (MSNE) difficult. This highlights the potential benefit of a classroom experiment that illustrates the intricacies of determining mixed strategies.

Classroom games are used to illustrate many game theory concepts, like the prisoner's dilemma (Holt and Capra) and Cournot and Bertrand games (Beckman). Rubinstein (1999) wrote an article describing the dozens of experiments he implemented in his undergraduate game theory class. Classroom games that address MSNE are limited, however, with the exception of Reiley, Urbancic, and Walker (2006). In their experiment, participants played a simplified version of poker that introduced a number of topics, including signaling, Bayes' Rule, bluffing, the value of information, and mixed strategies.

In this classroom experiment, students participate in a football play-calling experiment. This paper provides value above the previous game that touches on mixed strategies in that it focuses on several concepts relating to the MSNE. There is no pure-strategy Nash equilibrium in this game, so students are forced to think about why they must randomize their actions in equilibrium.[2] This game also provides students the opportunity to see, quite clearly, how they should deviate from the MSNE solution when their opponent is not playing optimally. The discussion following the experiment allows students a greater understanding of these concepts. The experiment is designed for several potential audiences. It is appropriate for undergraduate students taking a game theory course. It is also appropriate for students in an upper-level undergraduate microeconomics course that covers game theoretic concepts. The experiment is simple enough to understand that it can also be appropriate for a principles of microeconomics course, if only to give students an easy to understand (and fun) introduction to an advanced game theory concept.

THE EXPERIMENT

Students in this game play the role of either an offensive or defensive football coach. The offensive and defensive coaches compete against each other by playing a zero-sum game. The goal in this game for the offense is to gain yards while the goal of the defense is to prevent the offense from gaining yards. At the end of the experiment, the offense is given miniature candies based on the yards they gained – more yards gained means more candies; while the defense is given miniature candies based on the yards they give up – fewer yards allowed means more candies. I give students the option of selecting any of three candies: any candies could be used.

Students are randomly assigned to play the role of offense or defense and then are told to find someone who plays the opposing role. I choose different colors for the packets, e.g. the offense uses a plain-colored packet while the defense has a multi-colored packet. This makes it easy for students to find someone with the opposing role. The packet for the offense is in the appendix.[3] The choices of the game are simple for each side: the offense must choose whether they will run or a pass, while the defense must choose whether to defend the run or defend the pass. The yards an offense (defense) will gain (give up) are shown in the payoff matrix in figure 1. Students are shown the payoff matrix and the payoffs are also explained orally before any decisions are made. Given these payoffs, the offense will achieve the highest payoffs by choosing differently than the defense (i.e., choosing to run when the defense defends the pass or choosing to pass when the defense defends the run), while the defense will achieve the highest payoffs by defending the against the play the offense is choosing (i.e., choosing to defend the run when the offense chooses to run or choosing to defend the pass when the offense is passing).

There are three sessions to this game. In the initial session, the offense and defense choose whether to run or pass (or defend the run or pass) for each of the nine plays simultaneously (and confidentially). The students make choices for nine plays to help

Figure 1. Payoff Matrix for the Game

(Offensive payoff in terms of yards gained, Defensive payoff in terms of yards given up)

		Defense prepared to stop	
		Run	Pass
Offensive play	Run	(1,1)	(5,5)
	Pass	(7,7)	(2,2)

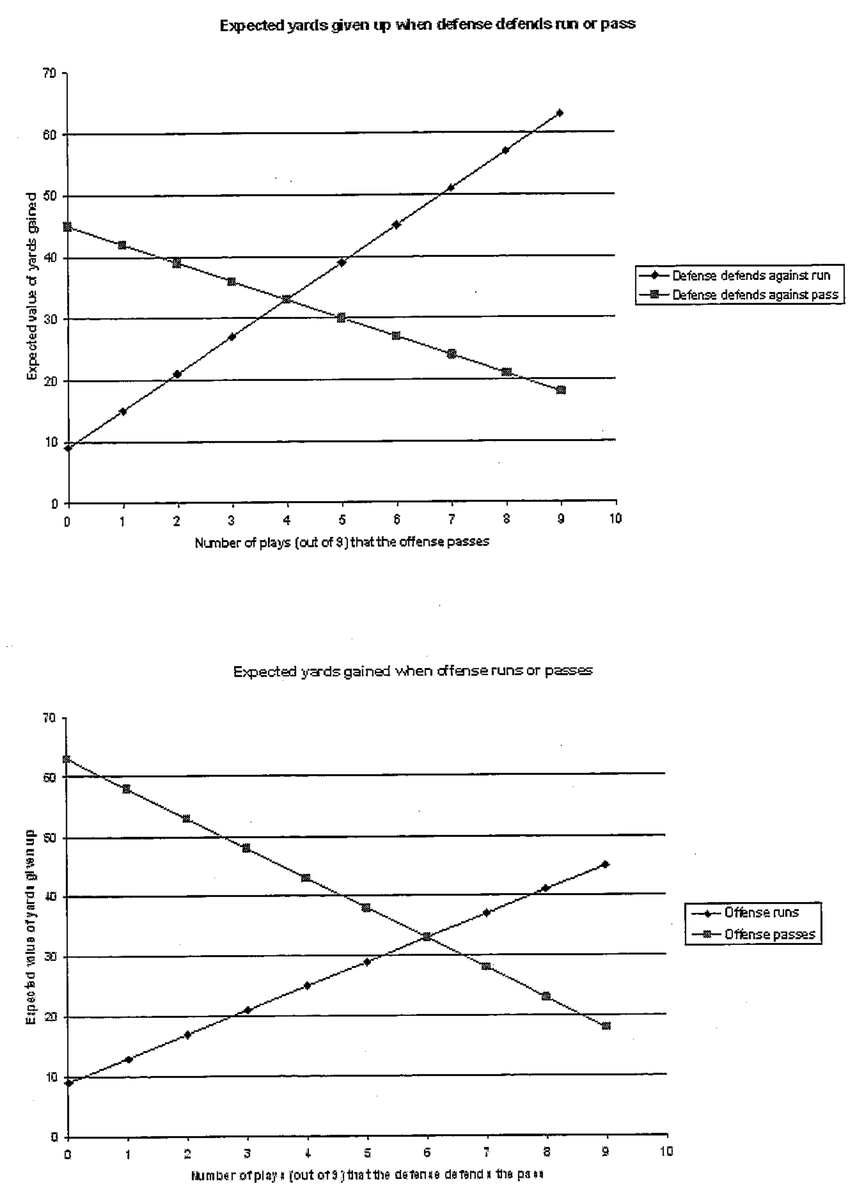
better illustrate the idea that they should consider randomizing their actions. With these payoffs, the mixed strategy Nash equilibrium is for the offense to pass 4/9 of the time and run 5/9 of the time while the defense should defend against the pass 2/3 of the time and defend against the run 1/3 the of the time. (See figure 2 for a graphical representation of the MSNE or figure 3 for its algebraic derivation.) However, untrained students will (usually) not know this. Further, many students will play a guessing game based on what they "think" their opponent in the game will choose. After the students make their choices, the coaches share their choices and determine the number of yards gained (given up) for each of the nine plays and then add up the yardage. Both the offense and defense note this in their packet.

After this initial session, some students will have performed better than others, but it may not be apparent to some students that it is optimal to mix strategies, or how strategies should be mixed. For example, some students may pick the same play every time but do very well because their opponent also picks the same play every time. (e.g. an offense chooses to pass every play and does well because the defense chooses to prevent the run every play). To help show the students the importance of mixing their strategies, there are two additional sessions in this classroom game. The second and third sessions help emphasize the importance of randomization.

In the second session, the offensive coach must (truthfully) tell the defensive coach how many times out of nine she will choose a running play and how many times out of nine she will choose a passing play. The instructor should emphasize that the offensive coach should not tell the defensive coach the exact plays he/she will run or pass. For example, if the offensive coach indicates he/she will run 3 times, the defensive coach should not be told in which of the 9 plays the offense will run. After the offensive coach gives the defensive coach this information, both coaches make their choices for each of the nine plays. In this session, most offensive coaches realize the need for randomization prior to making their announcement to the defense. After the students make their choices in session 2, they determine the number of yards gained (given up) for each of the nine plays and then add up the yardage.

The third and final session is similar, although this time the defensive coach is

Figure 2. Graphical Depiction of the Mixed Strategy Nash Equilibrium



required to (truthfully) tell the offensive coach how many times out of nine he/she will defend against the run and how many times out of nine he/she will defend against the pass. Once again, this is a point where the instructor should emphasize that the exact plays where the defense defends the run and pass should not be revealed, just the number of times out of nine that the run is defended. After the defensive coach gives the offensive coach this information, both coaches choose to run or pass for each of the nine plays. The second and third sessions highlight to students the need for randomization in games with no pure strategy Nash equilibrium. Astute students may even realize that it is correct to choose the run and pass approximately 50% of the time, even though the students have not been taught how to solve for MSNE.

DISCUSSION AND ASSESSMENT

The class discussion examines the choices the students made, how the difference in yards that can be gained by running and passing affects the students' choices, what the students perceive to be the optimal strategy, and the results. The students are quick to point out that the offense can gain more yards by passing, so many students tend to think that the optimal strategy (against an defensive coach playing optimally) for an offense is to pass more often than run with this game. This is incorrect. The students have a chance to discuss, on a non-technical level, that the offense should run slightly more than pass to keep the defense indifferent. The professor can elaborate by graphing the expected yards gained based each player's choices (figure 2) or by deriving the MSNE algebraically (figure 3).

The discussion also allows for an easy look at other topics that often confuse students. While learning a mixed strategy Nash equilibrium, it is useful to emphasize that the MSNE is the correct strategy only against an opponent that is playing optimally. If one finds herself in a game against an opponent that is playing sub-optimally, that opponent can usually be exploited. In this game, for example, participants can easily see that if an offense chose to run every time – the defense can simply choose to stop the run every time to maximize their expected value. These points can be discussed even to students who have not learned anything about mixed strategies. I have found that running this experiment shortly after teaching students about pure strategy Nash equilibrium works well as it provides students an easy example to relate back to while later working through the more technical details of computing the MSNE.

Finally, the discussion allows students a glimpse into an important feature of MSNE: that the correct mixed strategy won't always result in the expected outcome. In all games/situations where one plays mixed strategies, the MSNE will ensure the highest expected value, but the outcome could be much worse than the expected value. This allows discussion of real life situations where a player makes optimal choices but ends with the wrong outcome (e.g. in business decisions, political campaigns). If a professor wishes to avoid this topic, however, one could simply write a computer pro-gram that runs the game an infinite number of times to get to the long-run equilibrium

Figure 3. Calculating the Mixed Strategy Nash Equilibrium Algebraically

What should the defense do?

In order to avoid being exploited by an astute offense, the defense should choose a proportion of defending the run and the pass that makes the offense indifferent between running and passing. Therefore, their job is to make:

Expected yards gained from passing = Expected yards gained from running.

Setting p = probability that the defense defends the pass, we have:

$$p \times 2 + (1 - p) \times 7 = p \times 5 + (1 - p) \times 1$$
$$p \times 2 + 7 - p \times 7 = p \times 5 + 1 - p \times 1$$
$$6 = p \times 9$$
$$p = 2/3$$

Thus, the MSNE calls for the defense to defend the pass 2/3 of the time and defend the run 1/3 of the time.

What should the offense do?

In order to avoid being exploited by an astute defense, The offenses goal is to make the defense indifferent between defending the run and pass. Therefore, their job is to make:

Expected yards yielded from defending the pass = Expected yards yielded from defending the run.

For the offense, set q equal to the probability the offense passes:

$$q \times 7 + (1 - q) \times 1 = q \times 2 + (1 - q) \times 5$$
$$q \times 6 + 1 = q \times 3 + 5$$
$$q \times 9 = 4$$
$$q = 4/9$$

Thus, the MSNE calls for the offense to pass 4/9 of the time and run 5/9 of the time.

expected value for each player.

To assess the impact of the experiment on student learning, I ran the experiment with two principles-level courses and gave a short (extra-credit) quiz both the day before and the day after running the experiment (no discussion of the quiz in-between). The quiz had a game with no pure-strategy Nash equilibrium and asked students two questions: 1) What is the Nash equilibrium (if one exists), and 2) what is the optimal strategy for each player in the game. The mean score for the pre-experiment quiz was 1.5 out of 4, while the mean score on the post-experiment quiz was 2.3 out of 4.

POPTENTIAL LIMITATIONS

There are three minor limitations with this experiment. First, because this study involves pairs of students playing against each other, having an odd number of students will provide a minor complication. I have worked around this issue in the past by having one offense play against two defenses. However, that offense only calculated its payoffs vs. one of the two defenses. Second, unfamiliarity with American football may cause some students to initially feel uncomfortable about the experiment. However, instructors should emphasize at the beginning that knowledge of football is not necessary or even useful; students simply need to know how to interpret the payoff matrix and that more yards is better for the offense and worse for the defense. A third limitation could occur in the price of the candy. The payoffs were constructed so that each student will win an average of 10 pieces of candy. This could be easily changed, however, should the initial cost be prohibitive. For example, if you change the starting number of candies for the defense to 10 and then make the offense gain/defense lose one candy per twenty yards gained, then the expected candies won will be five per person.

CONCLUSION

In this classroom experiment, students play a simple game to help foster their understanding of the mixed strategy Nash equilibrium. This simple exercise of calling running or passing plays highlights three key features of MSNE: that you must randomize your actions against an optimal player, there is an optimal percentage to randomize between running and passing, and that against a sub-optimal player, such randomization is unnecessary. With the experiment and follow up discussion, students should have a good intuitive grasp on MSNE, which can be helpful when teaching the technical details of the mixed strategy Nash equilibrium.

ENDNOTES

[1] The author thanks Jay Corrigan and two anonymous referees for helpful comments.

[2] A Nash equilibrium exists in almost all games under fairly broad circumstances. Specifically, a Nash equilibrium (either pure or mixed) exists as long as there are a finite

number of choices, and that the a choice set is non-empty, convex, and compact as well as having a continuous utility function that is quasiconcave (Mas-Colell, Whinston, and Green).

[3] An electronic copy of the instructions for both the offense and the defense can be obtained from the author upon request, or from the author's website at http://www.susqu.edu/facstaff/r/rousu/research/.

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Appendix A. Instructions Handed to Students - Offense

NAME: ____________________

Instructions for the Football Play-Calling Experiment - Offense

You are a football coach and your goal is to obtain as many yards as possible on this particular drive. You have two options: to run or to pass. Likewise, your opponent, the defense can prepare to stop the run or stop the pass.

The payoff matrix for this game is as follows: (Offensive payoff in terms of yards gained, Defensive payoff in terms of yards given up)

		Defense prepared to stop	
		Run	Pass
Offensive play	Run	(1,1)	(5,5)
	Pass	(7,7)	(2,2)

Your goal as the coach controlling the offense is to gain as many yards as possible. You will choose to run or pass in order to do this. You will have 3 opportunities to play this game, with each opportunity having 9 separate plays. For every 10 yards you gain in this experiment, you will receive 1 miniature candy of your choice.

SESSION 1:

Please find a person in class who is a defensive coach in this experiment. Once you find an opponent, both of you will individually make decisions on the plays you will choose.

There are 9 plays – you must decide whether the offense should run the ball or pass the ball in each of these 9 plays (i.e., fill in column 2). Please make your choices now. DO NOT SHARE YOUR CHOICES WITH YOUR OPPONENT UNTIL AFTER YOU HAVE BOTH MADE YOUR DECISIONS

Play #	Your decision: run or pass?	Your opponent's decision: defending the run or pass?	Yards gained
1			
2			
3			
4			
5			
6			
7			
8			
9			
		TOTAL YARDS GAINED:	

Now that both you and your opponent have chosen your plays, you determine the yards gained for each play.

Who seemed to do better in this session, you or your opponent? Why?

SESSION 2:

For this session, you must tell the defensive coach how many times out of nine you will run and how many times out of nine you will pass. Your choices below MUST be consistent with the information you tell the defensive coach. Once you tell the defensive coach this information, you must determine whether you will run or pass on each of the nine plays. The defensive coach will simultaneously determine whether to stop the run or the pass on each of the nine plays. DO NOT SHARE YOUR CHOICES WITH YOUR OPPONENT UNTIL AFTER YOU HAVE BOTH MADE YOUR DECISIONS

Please make your choices now.

Play #	Your decision: run or pass?	Your opponent's decision: defending the run or pass?	Yards gained
1			
2			
3			
4			
5			
6			
7			
8			
9			
		TOTAL YARDS GAINED:	

Now that both you and your opponent have chosen your plays, you determine the yards gained for each play.

Who seemed to do better in this session, you or your opponent? Why?

SESSION 3:

For this session, the defensive coach must tell you how many times out of nine he/she will defend against the run and how many times out of nine he/she will defend against the pass. Once he/she tells you this, you must determine whether you will run or pass in each of the nine plays. The defensive coach will simultaneously determine whether to defend against the run or pass in each of the nine plays. DO NOT SHARE YOUR CHOICES WITH YOUR OPPONENT UNTIL AFTER YOU HAVE BOTH MADE YOUR

DECISIONS

Please make your choices now.

Play #	Your decision: run or pass?	Your opponent's decision: defending the run or pass?	Yards gained
1			
2			
3			
4			
5			
6			
7			
8			
9			
		TOTAL YARDS GAINED:	

Now that both you and your opponent have chosen your plays, you determine the yards gained for each play.

Who seemed to do better in this session, you or your opponent? Why?

Your total yards gained and candies earned:

Total yards gained in session	
1	
2	
3	
Total yards gained	
Candies earned (yards/10)	

Discussion questions:

Did you tend to choose to either run or pass more often? Why?

What would be the optimal strategy for an offense in this game?

How does the yardage you can gain by running vs. passing affect the optimal strategy?

What do you think the optimal strategy would be for the defense in this game?

Even if you choose optimally and your opponent does not, do you necessarily do better than your opponent? Why or why not?